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SELECTION OF A DESIRABLE EQUILIBRIUM BY SUBJECTIVE MOTIVE DISTRIBUTIONS

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1. INTRODUCTION

For a non-cooperative game Nash [5] proposes a concept of equilibrium which is well-known presently as the Nash equilibrium point (NEP). For simplicity we consider a two-person game. Let $M_i(x, y)$ be the expected payoff of player $i(=1, 2)$ when player 1 and 2 follow mixed strategies x and y respectively. The pair of strategies (x^*, y^*) for both players is a NEP if and only if the following relations hold.

$$M_1(x^*, y^*) = \max_x M_1(x, y^*) \tag{1}$$

$$M_2(x^*, y^*) = \max_y M_2(x^*, y) \tag{2}$$

The NEP is commonly used in the area of micro economics, but has the following problems :

(P 1) Many non-cooperative games have two or more NEPs. In such a case the NEP can not become a guiding principle for a player to select his desirable strategy.

(P 2) In many game experiments it is reported that even if the game has a unique NEP, many players do not use their Nash equilibrium strategy. For example, Minas et al. [3] reports that forty three percent of all subjects have used the pure strategy "cooperation" in the prisoner's dilemma game in which (defection, defection) is a unique NEP.

For the purpose of resolving the first problem (P 1), many investigations have been performed in two directions :

(a) The purification of equilibria : Selten [6] defines a subgame-perfect Nash equilibrium for a dynamic game with complete information. Harsanyi [1] defines a Bayesian Nash equilibrium for a static game with incomplete information. Furthermore a perfect Bayesian equilibrium is defined for a dynamic game with incomplete information.

(b) The selection of a desirable equilibrium : Selten and Harsanyi [2] try to structure a general theory with respect to the selection of a unique NEP in a non-cooperative game by introducing five concepts (1) the payoff dominance (2) the risk dominance (3) tracing procedure (4) isomorphism and (5) subgame consistency.

For the purpose of resolving the second problem (P 2), Nakai [4] proposes a subjective game. Considering subjective distributions on some motives, he explains the variety of strategies selected by players.

In this paper we propose a method of indicating a desirable strategy for a player by combining a subjective game with the payoff and risk dominances. We call this method by the SEMD method (the selection of an equilibrium by motive distributions). This method weeds out the less desirable NEPs by two criteria, the payoff dominance and the risk dominance, calculates probabilities of realization for all possible NEPs based on the subjective motive distributions, and finally asserts that it is desirable for a player to select the strategy indicated by the NEP having the maximum probability of realization. This method is not perfect, that is, it may occur that it can not select a unique NEP strictly, but this method is quite useful for many cases. We give some numerical examples for the explanation of the SEDM method. In one of them we can see that taking into account motive distributions, a point not being a NEP of the original game may be selected as a desirable point. Therefore we can explain the variety of selections of players. Thus we can somewhat resolve the above two problems (P 1) and (P 2) by the SEMD method.

2. THE "SEMD" METHOD

We consider a two-person non-cooperative finite game

$$G\ :\ \begin{array}{cc} & \text{player 2} \\ & \begin{array}{ccc} \beta_1 & \cdots & \beta_n \end{array} \\ \text{player1} \begin{array}{c} \alpha_1 \\ \vdots \\ \alpha_m \end{array} & \begin{pmatrix} (a_{11},b_{11}) & \cdots & (a_{1n},b_{1n}) \\ \vdots & & \vdots \\ (a_{m1},b_{m1}) & \cdots & (a_{mn},b_{mn}) \end{pmatrix} \end{array} \tag{3}$$

where $a_{ij}(b_{ij})$ denotes the payoff of player 1 (2), given that player 1 and 2 use pure strategies α_i and β_j respectively $(i=1,\cdots,m\ ;j=1,\cdots,n)$. We consider that each player selects his strategy under one of l motives $m_1,m_2,\cdots,m_l$. We can give some examples of motives as follows :

m_1 : maximization of his own payoff (selfish motive)

m_2 : maximization of the social payoff (the sum of both player's payoff) (coexistent motive)

m_3 : minimization of the opponent's payoff

m_4 : maximization of the winning probability

m_5 : maximization of the probability of not losing

m_6 : maximization of the difference between his own payoff and the opponent's payoff

Let $a_{ij}^s(b_{ij}^s)$ be the payoff of player 1 (2) when player 1 and 2 select the pure strategies α_i and β_j respectively and when player 1 (2) follow the motive m_s, for example,

$$\begin{array}{lll} a_{ij}^1 = a_{ij} & , & a_{ij}^2 = a_{ij} + b_{ij} \\ a_{ij}^3 = -b_{ij} & , & a_{ij}^4 = \operatorname{sgn}\left(a_{ij} - b_{ij}\right)^+ \\ a_{ij}^5 = 1 - \operatorname{sgn}\left(b_{ij} - a_{ij}\right)^+ & , & a_{ij}^6 = a_{ij} - b_{ij} \end{array} \tag{4}$$

where the symbol "sgn" denotes a sign function

$$\operatorname{sgn}\ x = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases} \tag{5}$$

and x^+ is the positive part of x, that is, $x^+ = \max\{x,0\}$.

We consider a certain player P (1 or 2) and assume that the player P thinks that player 1 follows a motive distribution $\lambda = \langle \lambda_1,\cdots,\lambda_l \rangle$ and that player 2 follows a motive distribution $\theta = \langle \theta_1,\cdots,\theta_l \rangle$ where $\lambda_s(\theta_s)$ is a probability that the player P thinks that player 1(2) follows the motive m_s. We define a subjective game of the player P by the bimatrix

$$G_p(\lambda,\ \theta) = \left[(\tilde{a}_{ij},\ \tilde{b}_{ij}) \,|\, 1 \le i \le m,\ 1 \le j \le n\right] \tag{6}$$

where $\tilde{a}_{ij} = \sum_{s=1}^{l} \lambda_s a_{ij}^s$ and $\tilde{b}_{ij} = \sum_{s=1}^{l} \theta_s b_{ij}^s$.

That is to say, the player P feels sure that he faces to his subjective game $G_p(\lambda,\ \theta)$.

Selten and Harsanyi [2] introduce concepts of the payoff dominance and the risk dominance.

DEFINITION 1. A NEP$(x^*,\ y^*)$ payoff dominates strictly another NEP $(\tilde{x},\ \tilde{y})$ if and only if

$$M_i(x^*,\ y^*) > M_i(\tilde{x},\ \tilde{y}) \quad \text{for } i = 1, 2 \tag{7}$$

which denotes that the NEP guaranteeing more expected payoffs to both players is desirable.

DEFINITION 2. A NEP $(x^*,\ y^*)$ risk dominates strictly another NEP $(\tilde{x},\ \tilde{y})$ if and only if

$$\begin{aligned} &\left\{M_1(x^*,\ y^*) - M_1(\tilde{x},\ y^*)\right\}\left\{M_2(x^*,\ y^*) - M_2(x^*,\ \tilde{y})\right\} \\ &> \left\{M_1(\tilde{x},\ \tilde{y}) - M_1(x^*,\ \tilde{y})\right\}\left\{M_2(\tilde{x},\ \tilde{y}) - M_2(\tilde{x},\ y^*)\right\} \end{aligned} \tag{8}$$

which denotes that the product of losses for players' deviations from the NEP $(x^*,\ y^*)$ is larger than the corresponding product from the NEP $(\tilde{x},\ \tilde{y})$. The inequality (8) means that both players consider that the NEP $(\tilde{x},\ \tilde{y})$ is more risky than the NEP $(x^*,\ y^*)$.

Next we propose the SEMD method indicating a desirable equilibrium to the player P. We put

$$K = \left\{(i,\ j) \,|\, 1 \le i \le l,\ 1 \le j \le l \quad i, j : \text{integer}\right\} \tag{9}$$

For any $(i,\ j) \in K$, let G_{ij} be the subjective game when player I and II follow motives m_i and m_j respectively. Furthermore let S_{ij} be the set of NEPs of the subjective game G_{ij}. We put

$$S = \bigcup_{(i,\ j) \in K} S_{ij}. \tag{10}$$

For any subjective game G_{ij} let $\tilde{S}_{ij}$ be the set of NEPs which are not payoff and risk dominated by other NEPs of the game G_{ij}. For a pair $(x,\ y)$ of strategies of both players in the game G_{ij} we introduce the following function:

$$I_{ij}(x,\ y) = \begin{cases} \left\|\tilde{S}_{ij}\right\|^{-1} & \text{if } (x, y) \in \tilde{S}_{ij} \\ 0 & \text{otherwise,} \end{cases} \tag{11}$$

where $\left\|\tilde{S}_{ij}\right\|$ is the number of elements of the set $\tilde{S}_{ij}$.

Let λ (θ) be the motive distribution of player 1 (2) which the player P thinks. We consider that when there are two or more NEPs, each NEP occurs with equal probability. Then the realization probability of NEP (x, y) for the player P is given by

$$p(x, y) = \sum_{(i, j) \in K} \lambda_i \theta_j I_{ij}(x, y). \tag{12}$$

Therefore we consider that the player P should aim at the realization of the NEP (x^*, y^*) having the maximum realization probability, that is,

$$p(x^*, y^*) = \max_{(x, y) \in S} p(x, y). \tag{13}$$

Summarizing the above discussion, we obtain the SEMD method as follows :

The SEMD method

Step 1 : We obtain all NEPs of all subjective games G_{ij} $(i, j = 1, \cdots, l)$.

Step 2 : For each G_{ij} we weed out a NEP which is payoff dominated by another NEP of G_{ij}.

Step 3 : For each G_{ij} we weed out a NEP which is risk dominated by another NEP of G_{ij}. (Note that the payoff dominance takes priority over the risk dominance.)

Step 4 : For each of the remained NEPs, we calculate its realization probability based on motive distributions λ and θ of both players which are subjective thought of the player P.

Step 5 : The player P should select the strategy indicated by the NEP having the maximum realization probability.

3. NUMERICAL EXAMPLES

In this section we show two numerical examples. Example 1 explains the application of the SEMD method for selecting one of three NEPs. Example 2 shows that even though the original game has a unique NEP, the selected equilibrium point may vary according to motive distributions of both players.

EXAMPLE 1. We consider a nonzero-sum game defined by Figure 1.

G : player 1		player 2 β_1	β_2
	α_1	2, 1	2, 3
	α_2	4, 1	1, 0

Fig.1 The game in Example 1

This game G has three NEPs, $A=(\langle 1,\ 0\rangle,\ \langle 0,\ 1\rangle)$, $B=(\langle 0,\ 1\rangle,\ \langle 1,\ 0\rangle)$, and $C=(\langle 1/3,\ 2/3\rangle,\ \langle 1/3,\ 2/3\rangle)$ at which the pairs of expected payoff of both players are given by $(2,\ 3)$, $(4,\ 1)$ and $(2,\ 1)$ respectively. None of them are payoff dominated by another one. We shall compare these NEPs from the viewpoint of risk dominance. Table 1 shows the pairs of expected payoff for strategies selected by both players. For example, if player 1 and 2 select strategies $\langle 1/3,\ 2/3\rangle$ and $\langle 1,\ 0\rangle$ respectively, the expected payoffs of player 1 and 2 are 10/3 and 1 respectively.

Table 1 The pairs of expected payoff

player2 / player1	$\langle 1,\ 0\rangle$	$\langle 1/3,\ 2/3\rangle$	$\langle 0,\ 1\rangle$
$\langle 1,\ 0\rangle$	2, 1	2, 7/3	2, 3 A
$\langle 1/3,\ 2/3\rangle$	10/3, 1	2, 1 C	4/3, 1
$\langle 0,\ 1\rangle$	4, 1 B	2, 1/3	1, 0

From Table 1 we know that the NEPs A and B risk dominate the NEP C and that there is no risk dominance relation between A and B. Then Nash equilibrium points can not be yet limited to a unique one. As criteria for selecting a strategy we consider two motives, m_1(selfish motive) and m_2(coexistent motive), which are defined in the previous section. Because $G_{11}=G$, the subjective game G_{11} has two realizable NEPs A and B, this is, $S_{11}=\{A,\ B\}$.

G_{12} : player 1		player 2 β_1	β_2
	α_1	2, 3	2, 5
	α_2	4, 5	1, 1

Fig.2 The subjective game G_{12}

The subjective game G_{12} is given by Figure 2 and has three NEPs A, B and $D=(\langle 2/3,\ 1/3\rangle,\ \langle 1/3,\ 2/3\rangle)$. The NEP D is payoff dominated by B. Moreover the NEP B risk dominates A. Thus G_{12} has a unique realizable NEP $B=(\langle 0,\ 1\rangle,\ \langle 1,\ 0\rangle)$, that is, $\tilde{S}_{12}=\{B\}$.

G_{21} : player 1 (rows), player 2 (columns)

	β_1	β_2
α_1	3, 1	5, 3
α_2	5, 1	1, 0

Fig.3 The subjective game G_{21}

Similarly the subjective game G_{21} is given by Figure 3 and has three NEPs A, B and $E=(\langle 1/3,\ 2/3\rangle,\ \langle 2/3,\ 1/3\rangle)$. The NEP E is payoff dominated by A, but there is no payoff dominance relation between A and B. Moreover the NEP A risk dominates B. Thus G_{21} has a unique realizable NEP $A=(\langle 1,\ 0\rangle,\ \langle 0,\ 1\rangle)$, that is, $\tilde{S}_{21}=\{A\}$.

G_{22} : player 1 (rows), player 2 (columns)

	β_1	β_2
α_1	3, 3	5, 5
α_2	5, 5	1, 1

Fig.4 The subjective game G_{22}

The subjective game G_{22} is given by Figure 4 and has three NEPs A, B and $F=(\langle 2/3,\ 1/3\rangle,\ \langle 2/3,\ 1/3\rangle)$. The NEP F is payoff dominated by A and B, but there is no payoff dominance relation between A and B. Moreover the NEP F is risk dominated by A and B. Thus G_{22} has two realizable NEPs A and B, that is, $\tilde{S}_{22}=\{A,\ B\}$.

Now we consider a certain player P (1 or 2) and let $\langle\lambda,\ 1-\lambda\rangle$ $(\langle\theta,\ 1-\theta\rangle)$ be the motive distribution of player 1(2) that the player P thinks. That is to say, λ (θ) is a probability that the player P thinks that player 1 (2) follows the motive m_1. Then we can obtain the realization probability of each NEP as follows :

$$p(A)=\frac{1}{2}\lambda\theta+(1-\lambda)\theta+\frac{1}{2}(1-\lambda)(1-\theta)=\frac{1}{2}(1+\theta-\lambda) \tag{14}$$

$$p(B)=\frac{1}{2}\lambda\theta+\lambda(1-\theta)+\frac{1}{2}(1-\lambda)(1-\theta)=\frac{1}{2}(1-\theta+\lambda) \tag{15}$$

Then if $\lambda \begin{Bmatrix} > \\ < \end{Bmatrix} \theta$, then $p(A) \begin{Bmatrix} < \\ > \end{Bmatrix} p(B)$.

Therefore if player 1 is more selfish than 2, then the player P should aim to the NEP B. If the player P is player 1(2), he should select the pure strategy $\alpha_2(\beta_1)$. Thus considering subjective games we could limit many equilibrium points to a unique one.

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